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The optimal commodity tax system
as a compromise between two objectives

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**The optimal commodity tax system
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Abstract

Policy analysis in applied fields such as agricultural, trade, environmental and development policy is still often undertaken within a first-best, rather than a more realistic second-best framework. The present paper seeks to contribute to changing this state of affairs by providing an intuitive explanation of what determines the optimal tax system. It derives and interprets an optimal tax formula for an economy with many goods to explain the optimal tax system as reflecting a trade-off between, on the one hand, the objective of encouraging the supply of labour to the market and, on the other hand, the objective of limiting the distortion of the marginal rate of substitution between produced goods. It illustrates this insight by a quantitative general equilibrium model which does not impose separability between consumption and leisure. The analysis clarifies issues of normalisation and deepens the insight due to Corlett and Hague (1953) that goods should be taxed according to their complementarity with leisure.

Keywords: public economics, optimal taxation, rules of normalisation, quantitative model of optimal taxation, Antonelli elasticity of complementarity.

JEL Classification: H2

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1. Introduction

The theory of optimal commodity taxation addresses the question of what characterizes the solution to the government's problem of raising a given amount of revenue by commodity taxes when the instrument of differential lump-sum taxation is not available. Aside from the obvious relevance for tax policy, the theory has important implications for labour market, environmental, transport, educational, pension, agricultural, trade and development policies, and for a range of other policy areas involving government decisions on pricing and taxes.

The development of the modern theory of optimal commodity taxation was launched at the beginning of the 1970s by a number of articles on optimal taxation and public sector pricing by prominent theoretical economists¹ who after the end of the decade moved on to other fields leaving behind contributions where the insight into what determines the optimal tax system was somewhat obscured, in part due to ambiguous justifications of rules of normalisation. No intuitive appealing interpretation of what characterises the optimal tax system has subsequently emerged. A leading expert in the field in fact considers the theory of optimal taxation as “*quite impenetrable from an intuitive point of view*” and that it for that reason has had a rather limited political impact (Boadway 1997).

Policy analysis in applied fields is still often conducted within a first-best framework, assuming (explicitly or implicitly) that government expenditures and income redistribution can be financed by lump-sum taxation. As this assumption is blatantly unrealistic, it potentially compromises the policy relevance of such analyses. This is in particularly true in the context of trade policy and climate change policies. It has increasingly been realised that the international community and national governments, particularly in developing countries, face important challenges in terms of tax collection, income redistribution and provision of public goods which can only be understood appropriately within a second-best framework. It is therefore unfortunate that optimal tax theory, which provides a much more relevant framework for addressing these issues than the *Second Theorem of Welfare Economics*,² has had such a limited impact.

On this background we set out, on the one hand, through a non-technical analysis supported by a quantitative example, and, on the other hand, by the derivation of an optimal tax formula, to provide an intuitive insight into what determines the optimal tax system. In order to focus on this objective, we employ a standard optimal tax model with only one representative household. For the sake of exposition we thus deliberately

¹ Including Dixit (1970), Baumol and Bradford (1970), Diamond and Mirrlees (1971), Dasgupta and Stiglitz (1971), Stiglitz and Dasgupta (1972).

² Atkinson and Stiglitz in their 1980 textbook, which consolidated the first 10 years of contributions to the modern theory of optimal taxation, and which remains the most authoritative reference to this literature, write on p357 as a comment to the relevance of policy analysis based on the *Second Theorem of Welfare*, “*The basic difficulty is again that the information on which we would like to base differential lump-sum taxes is not observable, or is observable only at great cost, and individuals have an incentive not to reveal it. For these reasons, lump-sum taxes and transfers are widely assumed not to be available*”

have left out considerations of equity, external effects, international trade, intertemporal allocation of resources, production inefficiency and administrative costs, which it would be important to take into account in any application of optimal tax theory in practice.

The article is organised as follows: In *Section 2*, we identify unsolved theoretical issues which we address in the paper. In *Section 3*, we specify the general equilibrium tax model on which the analysis is based, and derive the standard results of optimal commodity taxation as the basis for the further analysis. In *Section 4*, we provide, supported by a quantitative example, intuitive insight into what determines the optimal tax system. In *Section 5*, starting from the standard conditions for optimality, we derive and interpret an optimal tax formula in terms of Antonelli elasticities of complementarity. In *Section 6* we discuss the implication of the insight provided by the analysis for policy analysis. A final section summarises and suggests directions for future research.

2. Unresolved issues

During the development of the theory of optimal taxation in the 1970s, the government's problem was at first formulated in terms of maximization of an indirect utility function subject to the government's budget constraint (see e.g. Myles 1995). Theoretical results were subsequently derived from alternative formulations of the government's problem using the direct utility function (Atkinson and Stiglitz 1980), the expenditure function (Diamond and McFadden 1974 and Dixit 1975) and the distance function (Deaton 1979, 1981)³. Although less convenient for deriving and interpreting analytical results than the expenditure function approach, the indirect utility function approach has remained standard in the optimal tax literature⁴.

Interpretation of formal results has been provided in terms of the Ramsey rule (Ramsey 1927, Diamond and Mirrlees 1976) (see below), but by Mirrlees and others emphasized the importance of addressing optimal tax problems within a general equilibrium framework not in terms of optimal tax formulae. The inverse elasticity rule, known from partial equilibrium analysis and interpreted as reflecting the Ramsey rule, has been derived within the general equilibrium framework, but only by imposing extremely restrictive assumptions (see e.g. Atkinson and Stiglitz 1980), and has thus been of little assistance for the understanding of what determines the optimal tax system in general.

However, many contributions, ostensibly within the Diamond and Mirrlees framework, did not spell out the general equilibrium assumptions for the analysis, thus obscuring

³ The link between *Indirect utility function approach* and the *Expenditure function approach* is well understood, but the link to the *Distance function approach* seems not to be.

⁴ This is in fact surprising. The theory of optimal taxation is closely related to trade theory where for this reason general equilibrium conditions for decades have been formulated using the expenditure function approach.

the theoretical foundation for the analysis and the rationale for the normalisation rules adopted.

The question of when a proportional tax system is optimal initially received different answers, partly due to lack of clarity with respect to the justification of the rules of normalization adopted. However, it was soon realised, that a proportional tax system, based on all commodities, including the supply of labour to the market, will raise no revenue and therefore is not feasible and that there is no theoretical justification for why the optimal tax system based on produced commodities should be expected to be proportional. Nevertheless, partly because the use in applied works of functional forms, such as the CES, which impose separability between consumption and leisure, it has become the received wisdom that the optimal tax system should be expected to be close to proportional. It therefore remains an open question if, particularly in developing countries with large informal sectors, this view is justified.

Dating back to the original contributions to the optimal tax literature there have been ambiguities with respect to the interpretation of rules of normalisation. Furthermore, theoreticians working within the Mirrlees' tradition have been reluctant to consider optimal tax rules as they were seen to depend on arbitrary rules of normalisation. As a consequence what determines the optimal commodity tax system is less well understood than could be the case. The textbook optimal tax formula for an economy with only two produced commodities has been interpreted in support of the Corlett and Hague (1953) insight that those commodities most complementary with leisure should be taxed at the highest rates, e.g. by Layard and Walters (1978). However, the tax formula has also been interpreted in terms of the complementarity with the untaxed numeraire rather than with leisure (see e.g. Dixit 1975), an interpretation also emphasised in a relatively recent, widely used textbook⁵. Whether the Corlett and Hague insight can be generalized to an economy with many commodities, and the importance, if any, of the complementarity with leisure for the optimal tax system therefore remained unclear.

⁵ Myles (1995, p124) writes “It has been shown that in an economy with constant returns to scale consumer and producer prices can be normalised separately and that standard procedure is to make one good the numeraire and set the consumer and producer prices equal. This normalisation also has the effect of setting the tax on that good to zero. In particular, the zero tax is just a result of the normalisation rule. In particular, the zero tax carries no implication about the nature of the good nor about the ability to tax that good. This follows since the good with zero tax can be chosen arbitrarily from the set of available goods. Unfortunately, this reasoning has not been as clearly appreciated in some literature, it has been inferred from this that, since leisure cannot be measured in the same way as purchase of other commodities can, the zero tax on leisure is a restriction on the permissible tax system brought about by an inability to tax leisure. In addition the further inference is usually made that the optimal tax system aims to overcome this missing tax on the leisure by taxing goods complementary to leisure. Particular examples of this is found in the Corlett and Hague (1953) by ‘taxing those goods complementary with leisure, one is to some extent taxing leisure itself’ (p. 26) and Layard and Walters (1978) ‘the theory of second best tells us that if we cannot tax leisure we can do better than by taxing all goods equiproportionally’ (p. 184). Many other instances of similar statements could easily be given. This of course is a false interpretation. When real restrictions upon the permissible range of tax instruments are introduced the result obtained are affected. A number of such restrictions are considered in Munk (1980) where it is shown that the resulting optimal tax structure is sensitive to the precise restrictions imposed”.

3. The standard general equilibrium tax model

In this section we spell out in some detail the general equilibrium foundation for optimal tax analysis. This is important in order to deal correctly with the issue of normalisation and also in order to establish the links between different approaches to optimal tax analysis.

We consider a competitive economy with one household and a government where N commodities are produced by one primary factor. The primary factor is labelled 0 , and the N produced commodities are labelled $1, \dots, N$. We denote the set of commodities FC and the set of produced commodities C . The household's vector of endowments is $\omega \equiv (\omega_0, 0, \dots, 0)$ and its vector of consumption $\mathbf{c} \equiv (c_0, c_1, \dots, c_N)$. The vector of market transactions by the household is thus $\mathbf{x} \equiv (x_0, x_1, \dots, x_N) \equiv \mathbf{c} - \omega$. The primary factor can be thought of as "time", making x_0 the supply of labour to the market measured negatively, and c_0 consumption of time for household production and relaxation, which we referred to as leisure following the convention in the literature⁶. Consumer prices are $\mathbf{q} \equiv (q_0, q_1, \dots, q_N)$ and producer prices are $\mathbf{p} \equiv (p_0, p_1, \dots, p_N)$. The government's resource requirements is $\mathbf{x}^G \equiv (x_0^G, x_1^G, \dots, x_N^G)$ and its expenditures thus $G \equiv \mathbf{p}'\mathbf{x}^G$. The household's preferences are represented by a strictly quasi-concave utility function $u(\mathbf{x})$. Production possibilities are represented by constant returns to scale production functions with y_i being the output of commodity i and with y_0^i being the amount of the primary factor used in its production. The government's expenditures are financed solely by commodity taxes, $\mathbf{t} \equiv \mathbf{q} - \mathbf{p}$.

For such a tax system to be feasible, the three basic conditions for a market equilibrium, *Profit maximisation*, *Utility maximisation* and *Material balance* have to be satisfied, as well as the condition for the government's budget to be balanced.

Profit maximization implies that

$$y_0^i = -a_0^i y_i \quad i \in C \quad (1)$$

$$p_i = a_0^i p_0 \quad i \in C \quad (2)$$

The zero profit condition (2), implies that the value of the aggregate profit function

$$\sum_{i \in C} \Pi^i(p_0, p_i) \text{ is identically equal to zero.}$$

Following Dixit (1975) we represent the condition for utility maximisation by

$$\mathbf{x} = \mathbf{E}_{\mathbf{q}}(\mathbf{q}, u), \quad (3)$$

⁶ The term "leisure" is in fact misleading; a better term without misleading connotations is "the household's consumption of labour".

$$E(\mathbf{q}, u) \leq I, \quad (4)$$

where $E(\mathbf{q}, u)$ is the expenditure function corresponding to the household's utility function.⁷ The first of these two equations (3) says that \mathbf{x} must be the solution to the household's problem of minimising the expenditures required to achieve the utility level u at the prices \mathbf{q} ; the second equation (4) says that the household's unearned income I must be larger or equal to these expenditures. By the zero profit condition the household receives no unearned income, i.e. $I = 0$.

Material balance for goods and labour, respectively, are

$$y_i = x_i + x_i^G, \quad i \in C \quad (5)$$

$$\sum_{i \in C} y_0^i = x_0 + x_0^G. \quad (6)$$

The government's budget constraint is

$$\mathbf{t}' \mathbf{E}_q(\mathbf{q}, u) = G, \quad (7)$$

By successive substitutions, equations (1) to (7) may be reduced to

$$\sum_{i \in C} a_0^i (E_i(\mathbf{q}, u) + x_i^G) + E_0(\mathbf{q}, u) + x_0^G = 0, \quad (8)$$

$$E(\mathbf{q}, u) = 0, \quad (9)$$

$$(q_0 - p_0) E_0(\mathbf{q}, u) + (q_i - a_i p_0) \sum_{i \in C} E_i(\mathbf{q}, u) = G. \quad (10)$$

By Walras' law we can delete either (8) or (10). First deleting the government's budget constraint (10), from the remaining general equilibrium conditions (8) and (9) it follows by the homogeneity of $E(\mathbf{q}, u)$ and $\mathbf{E}_q(\mathbf{q}, u)$ in \mathbf{q} of degree one and degree zero, respectively, that the value of one consumer price can be fixed as a matter of normalisation. As furthermore by the zero profit condition an equilibrium is unaffected by the choice of the producer price level, we can also as matter of normalisation assume one producer price as fixed. In the optimal tax literature it is therefore often assumed that "labour is untaxed". However, to avoid misunderstandings it is important to distinguishing between the assumption that "the supply of labour to the market x_0 is untaxed", which an arbitrary rule of normalisation, and that "the household's

⁷ $E(\mathbf{q}, u)$ is the value function corresponding to the problem $\left\{ \min_{\mathbf{x}} \mathbf{q}' \mathbf{x} \text{ s. t. } u = u(\mathbf{x}) \right\}$. Using the subscript notation, we write net demand functions as $\mathbf{E}_q(\mathbf{q}, u) \equiv \{E_i(\mathbf{q}, u), i \in FC\} = \{x_i(\mathbf{q}, u), i \in FC\}$ and the corresponding partial demand derivatives as $\mathbf{E}_{qq}(\mathbf{q}, u) \equiv \{E_{ij}(\mathbf{q}, u), i, j \in FC\} = \left\{ \frac{\partial x_i}{\partial q_j}(\mathbf{q}, u), i, j \in FC \right\}$

consumption of labour (leisure) c_0 cannot be taxed”, which is a behavioural assumption.⁸

Deleting the material balance condition (8), the general equilibrium conditions for a tax vector \mathbf{t} to be feasible may be expressed as

$$E(\mathbf{p} + \mathbf{t}, u) = 0, \quad (11)$$

$$\mathbf{t}' \mathbf{E}_q(\mathbf{p} + \mathbf{t}, u) = G. \quad (12)$$

Solving (11) for u and then substituting in (12) the general equilibrium conditions may in fact be further reduced to just one equation

$$\mathbf{t}' \mathbf{x}(\mathbf{p} + \mathbf{t}, 0) = G, \quad (13)$$

where $\mathbf{x}(\mathbf{p} + \mathbf{t}, I)$ is the vector of ordinary demand functions.

Adopting the *Expenditure function approach* imposing (11) and (12) as constraints on the government's choice of tax instruments (rather than (13) as under the *Indirect utility function approach*) the government's maximisation problem may be formulated as⁹

$$\begin{aligned} & \underset{\mathbf{t}, u}{Max} \quad u \quad \text{s.t.} \\ & E(\mathbf{p} + \mathbf{t}, u) = 0 \\ & \mathbf{t}' \mathbf{E}_q(\mathbf{p} + \mathbf{t}, u) = G. \end{aligned} \quad (14)$$

The corresponding Lagrangian expression is

$$\mathcal{L} = u + \mu(-E(\mathbf{p} + \mathbf{t}, u)) + \lambda \left(\sum_{i \in FC} t_i E_i(\mathbf{p} + \mathbf{t}, u) - G \right). \quad (15)$$

The first order conditions with respect to u and $\mathbf{q} \equiv (q_0, q_1, \dots, q_N)$, for an optimal solution are, respectively,

$$\frac{\partial \mathcal{L}}{\partial u} = 1 - \mu E_u + \lambda \sum_{i=0}^N t_i E_{iu} = 0, \quad (16)$$

$$\frac{\partial \mathcal{L}}{\partial q_k} = -\mu x_k + \lambda \left(\sum_{i \in FC} t_i E_{ik} + x_k \right) = 0, \quad k \in FC \quad (17)$$

where $E_u \equiv \frac{\partial E}{\partial u} = 1 / \frac{\partial V}{\partial I}(\mathbf{q}, I)$, $E_{iu} \equiv \frac{\partial^2 E}{\partial q_i \partial u} = \frac{\partial x}{\partial I}(\mathbf{q}, I) / \frac{\partial V}{\partial I}(\mathbf{q}, I)$.

⁸ On this point we differ from Dixit (1975) who is in error when assuming that also c_0 can be assumed untaxed as a matter of normalisation.

⁹ Under the *Indirect utility function approach*, which has become the standard in the optimal tax literature, the government's maximisation problem may alternatively be formulated as $\underset{\mathbf{t}}{Max} V(\mathbf{t} + \mathbf{p}, 0)$ s. t. $\mathbf{t}' \mathbf{x}(\mathbf{p} + \mathbf{t}, 0) = G$. However, the *Expenditure function approach*, adopted here, has compared with the *Indirect utility function approach* advantages both in terms of ease of derivation and in terms of interpretation of results.

From (16) we then have that the *net social value of income* (see Diamond 1975) is

$$\mu = \alpha + \lambda \sum_{i=0}^N t_i \frac{\partial x_i(\mathbf{q}, I)}{\partial I}, \quad (18)$$

where $\alpha \equiv \frac{\partial V(\mathbf{q}, I)}{\partial I} = 1/E_u$.

Reordering (17) we obtain in matrix notation the conditions to be satisfied for an optimal tax system,¹⁰

$$\mathbf{E}_{\mathbf{q}\mathbf{q}} \mathbf{t} = -\theta \mathbf{x}, \quad (19)$$

where $\theta \equiv \frac{\lambda - \mu}{\lambda} > 0$ because the Slutsky matrix $\mathbf{E}_{\mathbf{q}\mathbf{q}} \equiv \{E_{ij}(\mathbf{q}, u), i, j \in FC\}$ is negative definite. In the Mirrlees tradition of optimal commodity taxation (see Mirrlees 1976), (19) is interpreted as the so-called *Ramsey rule*, that at the optimum the reduction in compensated demand for all commodities $\Delta x_i = E_i(\mathbf{q}^*, u^*) - E_i(\mathbf{p}, u^*)$, $i \in FC$ relative to the first best solution \mathbf{x}^* is approximately proportional. Applying this rule to commodity 0, we have

$$\frac{\Delta x_0}{x_0^*} \simeq \frac{\sum_{i \in FC} E_{0i}(\mathbf{q}^*, u^*) t_i}{x_0^*} = -\theta. \quad (20)$$

The optimal tax conditions (19) may therefore also be interpreted as saying that the basic distortion caused by the government being obliged to raise revenue by taxes based on market transaction rather than lump-sum taxation is that the supply of labour to the market is discouraged.

As pointed out by Boadway (1997), the Ramsey rule does not provide much insight into what constitutes desirable directions of tax reform, or what characterises the optimal tax system. However, for an economy with only two produced commodities, i.e. $FC = (0, 1, 2)$, an explicit optimal tax formula has been derived and interpreted in support of the Corlett and Hague conjecture that the commodity most complementary with leisure should be taxed at the highest rate¹¹. Assuming labour untaxed as a matter of normalisation, i.e. $t_0 = 0$, solving (19) for the optimal tax rates, it is possible to obtain (see e.g. Atkinson and Stiglitz 1980, pp375-376)

¹⁰ As is well known, this result can also be achieved using the *Indirect utility functions approach*, however with less ease of derivation and of interpretation of Lagrangian multipliers.

¹¹ Many theoreticians emphasising the importance of analysing optimal tax problems within a general equilibrium framework have focused on characterising the optimum with reference to quantities rather than prices with the argument that optimal tax rules depend on arbitrary rules of normalisation. Realising however that ranking of tax rates are independent of rules of normalisation, it does make sense to derive optimal tax formulae to seek insight into what determines this ranking.

$$\frac{t_1/q_1}{t_2/q_2} = \frac{-\varepsilon_{11} - \varepsilon_{22} - \varepsilon_{10}}{-\varepsilon_{11} - \varepsilon_{22} - \varepsilon_{20}}, \quad (21)$$

where $\varepsilon_{ik} \equiv E_{ik} / \frac{x_i}{q_k}$, $i, k \in FC$ are compensated demand elasticities.

To gain further insight we express this tax formula in terms of Allen-Uzawa elasticities of substitution, defined based on the full income expenditure function $M(\mathbf{q}, u)$ ¹² as

$$\sigma_{ij} \equiv \frac{M(\mathbf{q}, u) M_{ij}(\mathbf{q}, u)}{M_i(\mathbf{q}, u) M_j(\mathbf{q}, u)}, \quad i, j \in FC \quad (22)$$

On this basis complementarity may be defined as (see Stern 2004):

Definition 1: Two commodities i and j are complements (substitutes) for a given level of utility if an increase in the price of the j^{th} commodity increases (decreases) the quantity consumed of the i^{th} commodity (keeping the prices of all other commodities constant), i.e. if the Allen-Uzawa elasticity of substitution $\sigma_{ij} \geq 0 (< 0)$ ($i \neq j$).

Since $M_{ij}(\mathbf{q}, u) = E_{ij}(\mathbf{q}, u)$

$$\varepsilon_{ij} = a_j \sigma_{ij}, \quad i \in C, j \in FC \quad (23)$$

where $a_j \equiv q_j c_j / R$ is the share of consumption of commodity j in the household's full income $R = p_0 \omega_0$.

By the homogeneity of degree zero of $E_i(\mathbf{q}, u)$ we have that

$$\begin{aligned} \varepsilon_{10} + \varepsilon_{11} + \varepsilon_{12} &= 0, \\ \varepsilon_{20} + \varepsilon_{21} + \varepsilon_{22} &= 0, \end{aligned} \quad (24)$$

and thus that

$$\varepsilon_{11} + \varepsilon_{22} = -(\varepsilon_{12} + \varepsilon_{21}) - (\varepsilon_{10} + \varepsilon_{20}). \quad (25)$$

Substituting in (21) by (23) and (25), and since $\sigma_{ij} = \sigma_{ji}$, we have

$$\frac{t_1/q_1}{t_2/q_2} = \frac{(a_1 + a_2) \sigma_{12} + a_0 \sigma_{20}}{(a_1 + a_2) \sigma_{12} + a_0 \sigma_{10}}. \quad (26)$$

¹² $M(\mathbf{q}, u)$ is the value function corresponding to the problem $\left\{ \min_{\mathbf{c}} \mathbf{q}' \mathbf{c} \text{ s. t. } u = u(\boldsymbol{\omega} - \mathbf{c}) \right\}$. Using the subscript notation, we write gross demand functions as $M_{\mathbf{q}}(\mathbf{q}, u) \equiv \{M_i(\mathbf{q}, u), i \in FC\} = \{c_i(\mathbf{q}, u), i \in FC\}$ and the corresponding partial demand derivatives as $M_{\mathbf{q}\mathbf{q}}(\mathbf{q}, u) \equiv \{M_{ij}(\mathbf{q}, u), i, j \in FC\} = \left\{ \frac{\partial c_i}{\partial q_j}(\mathbf{q}, u), i, j \in FC \right\}$

The interpretation of this tax formula may be expressed as:

Proposition 1: *In an economy with two produced commodities and one primary factor, labour, the optimal tax system will be characterised by*

- a) *that commodity, which is most complementary with leisure in terms of the Allen-Uzawa elasticity of substitution, always being taxed at the highest rate, i.e. if at the optimum $\sigma_{20}/\sigma_{10} > 1$, then $\frac{t_1}{q_1} / \frac{t_2}{q_2} > 1$;*
- b) *for a given value of the elasticity of substitution between the two produced commodities, σ_{12} , the difference in tax rates is the greater, the greater the numerical value of σ_{20}/σ_{10} ;*
- c) *for a given values of σ_{20}/σ_{10} the difference is also the greater, the smaller is σ_{12} .*

In the next section we provide a more intuitive interpretation of this tax formula.

4. Intuitive insight into what determines the optimal tax system

The defining assumption of optimal tax theory is that the government must base taxation on market transactions as neither the household's endowment nor the household's consumption as such can be observed by the government. In this section we provide an intuitive, non-technical explanation of why a proportional tax system under this assumption must discourage the supply of labour and that the optimal tax system therefore represents a trade-off for the government between two basic objectives: to encourage the supply of labour to the market, and to limit the distortion of marginal rate of substitution between produced goods.

By similar arguments as in the discussion above of rules of normalisation, it follows from the general equilibrium conditions (8) and (9) that the imposition of a proportional tax system $T = q_j/p_j$, $j \in FC$ based on market transaction $x_i, i \in FC$ will leave the material balance conditions and the household's budget constraint unchanged, and therefore will generate no tax revenue. Any feasible tax system based on market transactions will therefore be distortionary in the sense that the marginal rate of substitution must differ from the marginal rate of transformation in production.

If counterfactually commodity taxation based on consumption of all commodities including leisure were possible, then a feasible tax system would have to satisfy the following conditions (compare with (8) and (9))

$$\sum_{i \in C} a_0^i (M_i(\mathbf{p} + \mathbf{t}, u) + x_i^G) - \omega_0 + M_0(\mathbf{p} + \mathbf{t}, u) + x_0^G = 0, \quad (27)$$

$$M(\mathbf{p} + \mathbf{t}, u) = p_0 \omega_0, \quad (28)$$

where $M(\mathbf{q}, u)$ is the full income expenditure function. By the homogeneity of $M(\mathbf{q}, u)$ and $\mathbf{M}_q(\mathbf{q}, u)$ in \mathbf{q} of degree one and degree zero, respectively, a tax system $\mathbf{t}^* = (T^* p_0, T^* p_1, T^* p_2, \dots, T^* p_N)$, where $T^* = \frac{1}{(1 - \tau^*)}$, based on the consumption of all goods, $c_i, i \in FC$ is equivalent to a tax on the household's full income $R = p_0 \omega_0$ at rate $\tau^* = \frac{G}{R}$, and therefore a first-best solution.

Now, consider a tax reform replacing the first-best tax system \mathbf{t}^* with a feasible proportional tax system based on the consumption of all produced commodities $\mathbf{t} = (0, Tp_1, Tp_2, \dots, Tp_N)$. Such a tax reform clearly would increase the consumption of leisure c_0 by decreasing the price of leisure relative to the prices of other commodities. However, as $c_i = x_i, i \in C$, such a tax system can also be interpreted as a proportional tax system based on market transactions $x_i, i \in FC$. As t_0 may be fixed as a matter of normalisation without loss of generality, we therefore have:

Proposition 2: A feasible proportional tax system $\mathbf{t} = (t_0, Tp_1, Tp_2, \dots, Tp_N)$ based on market transactions $x_i, i \in FC$ will discourage the supply of labour to the market compared with the first best allocation.

The basic distortion caused by the government raising tax revenue by commodity taxation based on market transaction rather than by lump-sum taxation may therefore be seen as the discouragement of the supply of labour to the market¹³.

Proposition 2 suggests that a tax reform involving replacing a proportional tax system by a non-proportional tax system where the tax rates on those goods which are highly complementary with leisure have been increased, and the tax rates for those, which are less so, have been decreased, will alleviate this basic distortion by increasing the supply of labour to the market, and hence increase welfare. However, differentiating tax rates obviously cannot increase welfare indefinitely as it creates another distortion: the marginal rates of substitution in consumption between produced commodities become more and more at variance with the marginal rates of transformation in production. In other words, as the distortion of the supply of labour is decreased, the distortion of the relative consumer prices of produced commodities is increased. The optimal tax system is achieved where the marginal gain in terms of encouragement of the supply of labour

¹³ As indeed follows from our interpretation of the Ramsey rule (see Equation (20)).

corresponds to the marginal loss in terms of distorting the marginal rates of substitution in consumption. We therefore have:

Proposition 3: The optimal tax system represent a compromise between two objectives:

Objective 1: to encourage the supply of labour to the market, $-x_0$,

and

Objective 2: to limit the distortion the marginal rate of substitution in consumption between produced goods, $\frac{dx_i}{dx_j}$, $i, j \in C$.

Taking σ_{20}/σ_{10} as an indicator of Objective 1 and σ_{12} as an indicator of Objective 2¹⁴ we see that Proposition 1 may be seen as a formal proof of Proposition 3 in the case of an economy with only two produced goods. However, this leaves open the question if it is possible to provide a formal proof of Proposition 3 also in the case of more than two produced goods. We return to this question in the Section 5.

To provide a quantitative illustration of Proposition 3, we consider an economy with one primary factor, *Labour*, and two produced goods, *Good 1* which is complementary with leisure and *Good 2* which is less so. The household's endowment of the primary factor is $\omega_0 = 390$ and the government's requirement of *Labour* is $x_0^G = 50$. By choice of units all producer prices are equal to unity, i.e. $(p_0, p_1, p_2) = (1, 1, 1)$. When the government resource requirement is financed by a lump-sum tax, the equilibrium net trade vector of the household is $\mathbf{x}^{FB} = (x_0, x_1, x_2) = (-150, 50, 50)$ and the household's consumption of *Labour* (leisure) is therefore $c_0 = 240$. We represent household preferences by a CES-UT¹⁵ utility function $u(x_0, x_1, x_2) = u(c_0^0, C(C_1(c_0^1, x_1; \sigma_1^0), C_2(c_0^2, x_2; \sigma_2^0); \sigma^1) \sigma^2)$ where $c_0^0 + c_0^1 + c_0^2 = \omega_0 + x_0$, and where $u(c_0^0, C; \sigma^2)$, $C(C_1, C_2; \sigma^1)$ and $C_i(c_0^i, x_i; \sigma_i^0)$, $i \in (1, 2)$ are CES functions.

¹⁴ The larger is σ_{20}/σ_{10} , the larger will be the increase in the supply of labour by a given increase in the differentiation of tax rates. The larger is σ_{12} , the larger will be the increase in the distortion of the consumption of produced commodities by a given increase in the differentiation of tax rates.

¹⁵ For further details about the CES-UT (Constant Elasticity of Substitution with explicit representation of the Use of Time), see e.g. Munk (1998), "Optimal support to low income households", EPRU Working Paper 1998-20, University of Copenhagen. The important property of the CES-UT utility function in the context of Proposition 3 is that it, in contrast to utility functions in general used in quantitative general equilibrium models, does not impose separability between the consumption of produced commodity and leisure. With an additive separable utility functions, as for example a nested CES function, there is no trade-off between Objective 1 and Objective 2, and the optimal tax system is therefore proportional whatever the choice of parameter values.

The parameter values are $\sigma_1^0 = 0.1$, $\sigma_2^0 = 0.1$, $\sigma^1 = 1$ and $\sigma^2 = 0.5$. The consolidated matrix of substitution elasticities in the first best situation corresponding to these parameter values is indicated in *Table 1*.¹⁶

Table 1: *Matrix of elasticities of substitution*

$$\begin{bmatrix} \cdot & \sigma_{10} & \sigma_{20} \\ \sigma_{01} & \cdot & \sigma_{12} \\ \sigma_{02} & \sigma_{21} & \cdot \end{bmatrix} = \begin{bmatrix} \cdot & 0.006 & 0.372 \\ 0.006 & \cdot & 1.621 \\ 0.372 & 1.621 & \cdot \end{bmatrix}$$

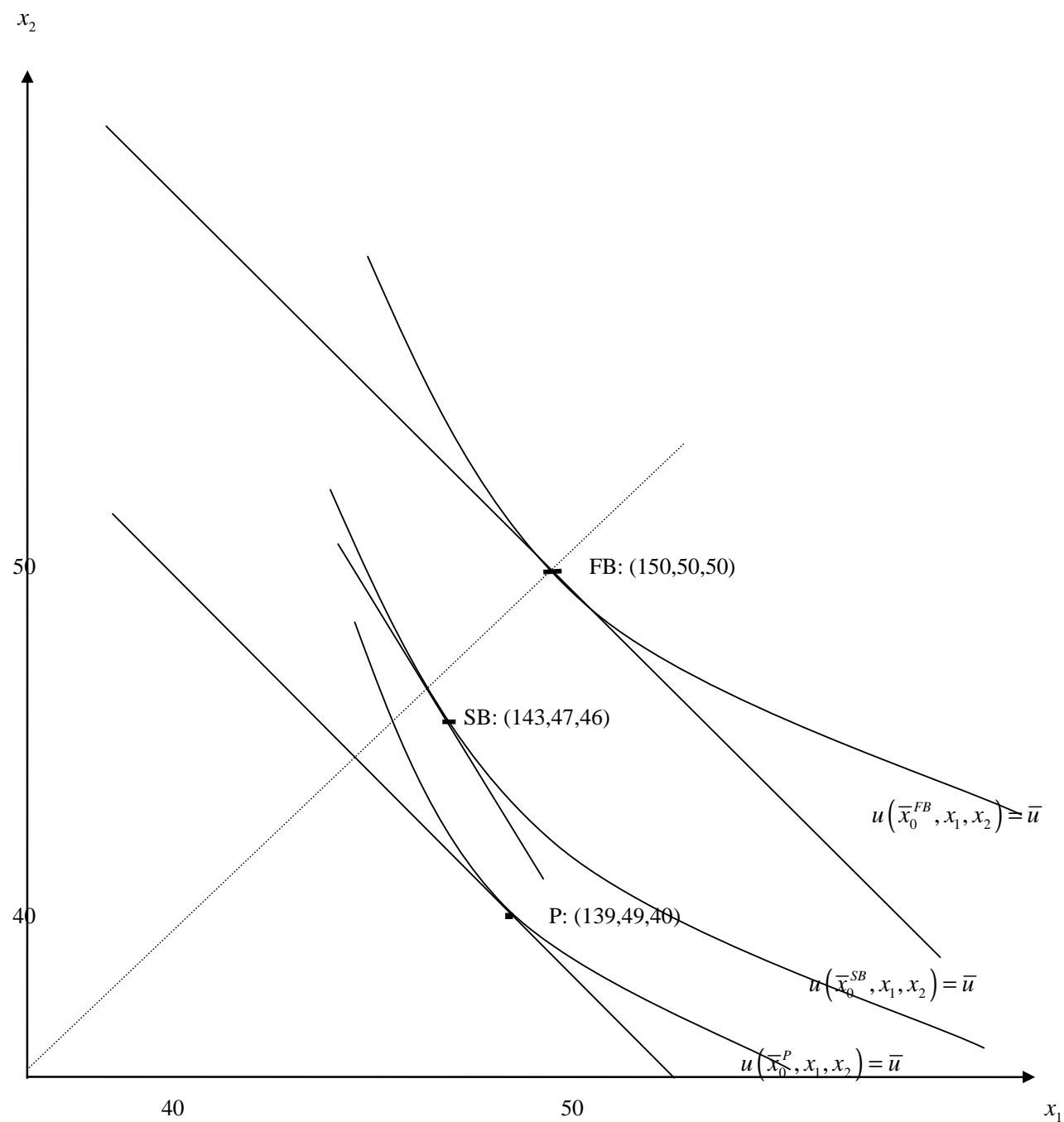
The equilibrium solution when the government's requirement is financed by a proportional commodity tax system is \mathbf{x}^P . The second- best optimal solution is \mathbf{x}^{SB} . As a matter of normalisation, we assume the supply of labour to the market as untaxed. For the benchmark data set and the parameter values indicated above, \mathbf{x}^P and \mathbf{x}^{SB} has been calculated by a small general equilibrium computer program. The results are provided in *Table 2*.

Table 2: *Market transactions under different tax systems*

	Lump-sum taxation FB	Proportional tax system P	Optimal commodity tax system SB
x_1	50	49	47
x_2	50	40	46
$-x_0$	150	139	143
q_1	1	1.56	1.77
q_1	1	1.56	1.35
$\frac{q_1}{q_2} = -\frac{dx_2}{dx_1}$	1	1	1.31

¹⁶ Notice that as $\sigma_{20} > \sigma_{10}$, Commodity 1 is more complementary with leisure than Commodity 2.

Figure 1: The First-Best (FB), Second-Best (SB) allocation and the allocation with a proportional tax system (PS).



In *Figure 1* the three equilibrium solutions are projected into the $x_1 - x_2$ space. The results indicate, first, that the supply of labour to the market is larger under an optimally differentiated than under a proportional commodity tax system, but smaller in the case of the first best solution, i.e. $-x_0^{FB} > -x_0^{SB} > -x_0^P$, and, second, that the second best solution represents a compromise between the two objectives as expressed in Proposition 3. If the difference in elasticities of substitution between consumption and leisure for the two produced goods σ_{20}/σ_{10} had been larger, or the elasticity of substitution between the two produced commodities σ_{12} had been smaller, then \mathbf{x}^{SB} would have involved a larger distortion of the rate of substitution and a larger supply of labour.

5. The optimal tax system in the case of many produced commodities

The fact that (26) cannot be generalised to an economy with more than two produced commodities suggests that the Allen-Uzawa elasticity of substitution does not provide the appropriate measure of complementarity for quantitative representation of Proposition 3, i.e. for understanding what determines the optimal tax system. We therefore consider an alternative measure of complementarity, the Antonelli elasticity. Using this measure we derive a tax formula similar to (26), but valid for an economy with many produced commodities.

By the homogeneity of degree 1 of $M(\mathbf{q}, u)$ we have

$$M(\tilde{\mathbf{q}}, u) = RM(\mathbf{q}, u), \quad (29)$$

where R is the household's full income and $\tilde{\mathbf{q}}$ is the vector of normalised commodity prices whose elements are $\tilde{q}_i = q_i/R$, $i, j \in FC$. Therefore

$$\tilde{\mathbf{M}}_{qq} = 1/R \mathbf{M}_{qq}. \quad (30)$$

The *distance function* may be defined as¹⁷.

$$D(\mathbf{c}, u) \equiv \min_{\tilde{\mathbf{q}}} \{ \tilde{\mathbf{q}} \mathbf{c} \text{ s.t. } M(\tilde{\mathbf{q}}, u) \geq 1 \}, \quad (31)$$

and the *Antonelli elasticity of complementarity* may be defined from the distance function (see Deaton 1979 and Stern 2004) as

$$\rho_{ij} \equiv \frac{D(\mathbf{c}, u) D_{ij}(\mathbf{c}, u)}{D_i(\mathbf{c}, u) D_j(\mathbf{c}, u)}, \quad i, j \in FC \quad (32)$$

Using this concept, complementarity may be defined as (see Stern 2004):

¹⁷ The *Distance function* indicates the amount by which the consumption vector must be deflated or inflated to reach the indifference curve associated with u . The Distance function is increasing, linear homogenous, and concave with respect to \mathbf{c} , and decreasing in u .

Definition 2: Two commodities i and j are complements (substitutes) for a given level of utility, if an increase in the consumption of the j^{th} commodity increases (decreases) the marginal valuation of the i^{th} commodity keeping the consumption of all other commodities constant, i.e. if $\rho_{ij} \geq 0 (< 0)$ ($i \neq j$).

The Antonelli matrix \mathbf{D}_{cc} is the generalised inverse of the Slutsky matrix $\tilde{\mathbf{M}}_{qq}$ with the implication that

$$\mathbf{D}_{cc}\tilde{\mathbf{M}}_{qq} = \mathbf{I} - \tilde{\mathbf{q}}\mathbf{c}'. \quad (33)$$

Starting from the well-known condition for an optimal tax system (19), we now derive an optimal tax formula for an economy with many produced commodities. Since $\mathbf{x} = \mathbf{c} - \boldsymbol{\omega}$ we have

$$\mathbf{E}_{qq}\mathbf{t} = -\theta(\mathbf{c} - \boldsymbol{\omega}). \quad (34)$$

Pre-multiplying by \mathbf{D}_{cc} , using that $\mathbf{M}_{qq} = \mathbf{E}_{qq}$ and that $\tilde{\mathbf{M}}_{qq} = (1/R)\mathbf{M}_{qq}$, we have

$$\mathbf{D}_{cc}\tilde{\mathbf{M}}_{qq}\mathbf{t} = -(\theta/R)\mathbf{D}_{cc}(\mathbf{c} - \boldsymbol{\omega}), \quad (35)$$

and then by (33) we have

$$(\mathbf{I} - \tilde{\mathbf{q}}\mathbf{c}')\mathbf{t} = -(\theta/R)\mathbf{D}_{cc}(\mathbf{c} - \boldsymbol{\omega}). \quad (36)$$

Assuming as a matter of normalisation that labour is untaxed, we obtain by similar derivations as in Deaton (1981) (see *Annex*)

$$\frac{t_k}{q_k} = \frac{G}{p_0\omega_0} \left(1 + \frac{\rho_{k0}}{\rho_{00}} \right). \quad k \in C \quad (37)$$

Dividing the condition for i with the condition for j we have

$$\frac{t_j}{p_j} \bigg/ \frac{t_i}{p_i} = \left(\frac{\rho_{00} + \rho_{j0}}{\rho_{00} + \rho_{i0}} \right). \quad j, i \in C \quad (38)$$

We therefore have:

Proposition 4: In an economy with many produced commodities and one primary factor, labour, the optimal tax system will be characterised by

- a) the commodity which is most complementary with leisure in terms of the Antonelli elasticities of complementarity being taxed at the highest rate, i.e. if at the optimum if $\rho_{i0} / \rho_{j0} > 1$ then $\frac{t_j}{q_j} \bigg/ \frac{t_i}{q_i} > 1$;
- b) for given values of ρ_{00} , the difference in tax rates is the greater, the greater the numerical value of ρ_{i0} / ρ_{j0} ;

c) for a given value of ρ_{i0} / ρ_{j0} , the difference is also the greater, the smaller the numerical value of ρ_{00} .

The size of ρ_{0j} represents the curvature of the indifference curve $u(x_0, \bar{x}_1, \bar{x}_2, \dots, x_j, \bar{x}_N) = \bar{u}$ and is thus related to the effect of increasing the price of j on the labour supply; the size of ρ_{00} represents the curvature of the indifference surface $u(\bar{x}_0, x_1, x_2, \dots, x_N) = \bar{u}$ and is thus related to the distortionary cost of differentiating the tax rates for produced commodities. Taking ρ_{i0} / ρ_{j0} as an indicator of the gain in terms of Objective 1, and ρ_{00} as an indicator of the loss in terms of Objective 2, Proposition 4 may be interpreted in support of Proposition 3, *i.e.* that the optimal tax system is determined as a trade-off between Objective 1 and Objective 2, where at the optimum the gain in terms of encouraging the labour supply is balanced by the loss due to a further distortion of the marginal rate of substitution between produced commodities.

Proposition 4 is valid for any number of produced commodities. The Antonelli elasticity of complementarity therefore constitutes the appropriate measure of complementarity for understanding what determines the optimal tax system. The Antonelli elasticity of complementarity between commodities i and j represents the curvature of the indifference curve where the consumption of all other commodities are kept constant, where the Allen-Uzawa elasticity of substitution assumes that the prices of all other commodities are kept constant. In the case of two produced commodities the question of keeping the amount of other produced commodities constant does not arise. This is the reason why in that case, *Proposition 1* and *4* provide basically the same insight into what characterises the optimal tax system.

6. Discussion of implications for applied policy analysis

In this section we consider the implications of the theoretical analysis for applied policy analysis.

The received wisdom in the public economic literature is that the potential welfare gains by differentiating tax rates are modest (see e.g. Heady 1993). Considering that the administrative costs are likely to be higher for a differentiated tax system than for a proportional tax system, it is indeed possible that the latter is the optimal solution to the government's problem of raising a given amount of tax revenue at minimum costs. However, when the complementarity between the consumption of purchased goods and leisure differs significantly (as for example between household services and leisure travel) and when the informal sector is large (as in many developing countries), a differentiated tax system may be associated with a significant welfare gain justifying the additional administrative costs.

We suggest that there are at least two reasons why the Corlett and Hague insight should be given more attention in the context of discussion of tax reform than is in general the case. *First*, the use of the term “leisure” in interpreting the Corlett and Hague insight is potentially misleading. It may give the false impression that it is the complementarity of the consumption of goods with the household’s use of time for relaxation which is important for the determination of the optimal tax system. In the context of the standard optimal tax model with one representative household and one primary factor, “leisure” in fact represents the use of labour not only for relaxation but for all consumption, production and exchange activities within the household sector (the informal sector) which the government cannot observe or can observe only at excessive administrative costs, and on which the government therefore cannot base taxation. In particular in developing countries where up to 80% of the labour resources are used in the informal sector stimulating the supply of labour for use in the formal sector by differentiating tax rates on household purchases of commodities produced in the formal sector may be associated with a considerable welfare gain. *Second*, during the last 50 years the share of government expenditures in national income has increased in most countries, increasing the distortionary costs of taxation, and at the same time the costs of tax administration have decreased due to innovations in information technology. Both developments have changed the trade-off between the allocative benefits and the administrative costs in favour of more complicated commodity tax systems. These insights seem important in relation to two areas which are currently prominent in the political debate: The use of domestic and border taxes to raise revenue in developing countries; and tax and price reform to combat global warming.

In developing countries with large informal sectors and with relative large costs of tax administration, the question has been raised if not border taxes would be better than domestic taxes to raise government revenue (see e.g. Emran and Stiglitz 2005, Gordon and Li 2005 and Munk 2008).

In environmental economics, under the heading of the “Double dividend”, it has been discussed if differentiation of taxes to achieve environmental objectives in addition to the environmental benefits would generate benefits in terms of allocational efficiency. The received wisdom among public finance economists is that there is no or little scope for a green tax reform to be associated with a double dividend (see eg. Goulder 1995, Bovenberg and de Mooij 1994, Bovenberg 1999 and Sandmo 2000). Indeed the case for discounting the possibility for a green tax reform being associated with a substantial double dividend may have been overstated by disregard of the administrative costs of taxation and the imposition of unrealistic separability assumptions. For example in road transport, a area where tax reform will be very important for combating both congestion and global warming, it has been suggested that the effect of higher taxes on transport in terms of stimulating the labour supply might be more significant than the benefits of reducing the external damage associated with transport (see Parry and Bonito 2001).

In summary, arguments in support of proportional tax systems which were valid when optimal tax theory was first developed in the 1970s with reference to conditions in

developed countries, may not any longer apply due to technological progress, and may never have been valid in developing countries due to their large informal sectors. Applied policy analysis need to accommodate this insight.

7. Summary and suggestions for future research

In this paper we have provided an intuitive explanation of what determines the optimal tax system as a trade-off between two objectives: 1) to encourage the supply of labour to the market and 2) to limit the distortion of marginal rates of substitution between produced commodities. We have derived a many-goods optimal tax formula and interpreted this and the well known optimal tax formula in the case of two goods in support of this insight. In this context we have identified the Antonelli elasticity as the appropriate measure of complementarity for understanding what determines the optimal tax system. This has provided an intuitive rationale and a formal proof for the insight first provided by Corlett and Hague (1953) that the goods most complementary with the household's use of labour ("leisure") should be taxed at the highest rates, and also for the fact that the differentiation should be the greater the more difficult the substitution between produced commodities. We have thus discarded the idea that the rule of normalisation adopted has any importance in this context. With respect to the implication for applied policy analysis, we have drawn attention to the potential importance in the context of tax reform analysis to use models with explicit representation of the consumption of time (leisure) in the household sector, and to avoid unrealistic separability assumptions with respect to the interaction between consumption and leisure.

On this background we suggest the following areas for future research: 1) empirical and theoretical analysis on how household consumption and production activities influence the preferences expressed by utility functions defined on the household's market transactions, and 2) empirical research on the costs of tax administration as a function of the design of the tax system, in particular in developing countries. The first suggests a need for further effort to combine theoretical insight from the theory of household production by Becker (1965) and Lancaster (1966) with optimal tax theory as pioneered by Atkinson and Stern (1979). The second suggests the need for combining the insight from the fast growing literature on the administrative costs of taxation (see e.g. Evan 2003) with optimal tax theory.

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Annex: Derivation of the optimal tax formula

We have

$$(I - \tilde{\mathbf{q}}\mathbf{c}')\mathbf{t} = -(\theta/R)\mathbf{D}_{cc}(\mathbf{c} - \boldsymbol{\omega}), \quad (39)$$

and thus that

$$\mathbf{t} = \tilde{\mathbf{q}}\mathbf{c}'\mathbf{t} - (\theta/R)\mathbf{D}_{cc}(\mathbf{c} - \boldsymbol{\omega}). \quad (40)$$

Since $\mathbf{D}_{cc}\mathbf{c} = 0$

$$\mathbf{t} = \tilde{\mathbf{q}}\mathbf{c}'\mathbf{t} + (\theta/R)\mathbf{D}_{cc}\boldsymbol{\omega}. \quad (41)$$

Substituting by the government's budget constraint, $(\mathbf{c} - \boldsymbol{\omega})'\mathbf{t} = G$, we get

$$\mathbf{t} = \tilde{\mathbf{q}}(G + \boldsymbol{\omega}'\mathbf{t}) + (\theta/R)\mathbf{D}_{cc}\boldsymbol{\omega}. \quad (42)$$

Multiplying by $\boldsymbol{\omega}'$ we have

$$\boldsymbol{\omega}'\mathbf{t} = \boldsymbol{\omega}'\tilde{\mathbf{q}}(G + \boldsymbol{\omega}'\mathbf{t}) + (\theta/R)\boldsymbol{\omega}'\mathbf{D}_{cc}\boldsymbol{\omega}. \quad (43)$$

Assuming as a matter of normalisation that labour is untaxed, i.e. $t_0 = 0$, and thus that

$\boldsymbol{\omega}'\mathbf{t} = 0$, we have

$$0 = \boldsymbol{\omega}'\tilde{\mathbf{q}}G + (\theta/R)\boldsymbol{\omega}'\mathbf{D}_{cc}\boldsymbol{\omega}. \quad (44)$$

The household's budget constraint is $\boldsymbol{\omega}'\tilde{\mathbf{q}} = 1$; thus

$$\theta/R = -\frac{G}{\boldsymbol{\omega}'\mathbf{D}_{cc}\boldsymbol{\omega}}. \quad (45)$$

Since $\boldsymbol{\omega} = (\omega_0, 0, \dots, 0)$

$$\omega_0 \theta/R = \frac{G}{D_{00}(\mathbf{c}, u) \omega_0}. \quad (46)$$

Substituting in (42) by (46), the necessary conditions for an optimal tax may be written as

$$t_k = G\tilde{q}_k + \frac{G}{D_{00}\omega_0} D_{0k}. \quad k \in C \quad (47)$$

Since $M = q_0\omega_0$ we obtain

$$t_k = \tilde{q}_k \left(G + \frac{D_{0k}/\tilde{q}_0\tilde{q}_k}{D_{00}/\tilde{q}_0\tilde{q}_0} \right). \quad k \in C \quad (48)$$

Substituting using the definitions of ρ_{ij} , we have

$$\frac{t_k}{q_k} = \frac{G}{R} \left(1 + \frac{\rho_{k0}}{\rho_{00}} \right). \quad k \in C \quad (49)$$

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